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MINIMUM ENERGY TEST DIRECTION DESIGN IN THE CONTROL OF CRYOGENIC WIND TUNNELS

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PREFACE

This report details the test direction planning problems associated with cryogenic wind tunnels, analyzed as a part of the project "Modeling and Control of Transonic Cryogenic Tunnels," sponsored by NASA/Langley Research Center (LaRC) under research grant NSG-1503. The report is concerned with realizing desired flow Reynolds number-Mach number combinations at which data is sought, with minimum liquid nitrogen consumption. The contents of this document complement the reports on modeling phase activity in reference 1 and the control analysis phase activity in reference 2.

MINIMUM ENERGY TEST DIRECTION DESIGN IN
THE CONTROL OF CRYOGENIC WIND TUNNELS

By

S. Balakrishna[1]

INTRODUCTION

Need for realization of full-flight Reynolds number flows past test models in relatively small wind tunnels was keenly felt in the 1970's. The advent of the cryogenic wind tunnel concept is directly attributable to this need for high Reynolds number flow in wind tunnels (ref. 3). The cryogenic wind tunnel concept consists of operating the test medium of a conventional tunnel at cryogenic temperatures down to 80 K. Nitrogen gas, cooled by injected liquid nitrogen, has proven to be an ideal candidate for the cryogenic tunnel test medium because of its near perfect behavior in insentropic flow (ref. 4). Further, cryogenic operation of a wind tunnel results in reduced fan power consumption and no penalty in flow dynamic pressure.

In a cryogenic tunnel, the flow parameters—Reynolds number, Mach number and flow dynamic pressure can be independently controlled by separately controlling the tunnel flow variables: total temperature, test-section mass flow, and the tunnel total pressure (ref. 3). The problem of closed-loop control of the tunnel total temperature, flow Mach number, and total pressure has been addressed and reported extensively in references 1 and 2. Utilizing the three tunnel control inputs, the liquid nitrogen valve area, the fan speed and the gaseous nitrogen bleed valve area, control strategies for precise closed-loop control of the tunnel variables have been derived and have been successfully demonstrated.

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Since aerodynamic coefficients of test models are sought and analyzed as functions of the flow parameters: Reynolds number R_e, flow Mach number M, and flow dynamic pressure q, a test program in a cryogenic tunnel obviously starts with the test-condition flow parameter set R_e, M, and q. Further, if the tests are not aeroelastic tests, the flow parameter set consists of R_e and M only. The translation of the test flow parameter set R_e-M-q or R_e-M into the cryogenic tunnel variables P-T is the problem of test direction design. In this research effort, this problem of test direction design has been addressed, on the basis of minimum consumption of liquid nitrogen for the whole test program.

In the foregoing analysis, a test direction parameter ϕ is defined. Its properties are used to derive ths order of test. Further, the problem of choosing the tunnel states P-T, such that minimum liquid nitrogen consumption occurs, is formulated and resolved.

NOMENCLATURE

a	velocity of sound in nitrogen (m/sec)
A	area (m^2 or m^2/m^2)
$\overline{c}$	chord (m)
C_p	specific heat of nitrogen gas at constant pressure (kJ/kg-K)
C_v	specific heat of nitrogen gas at constant volume (kJ/kg-K)
C_m	specific heat of tunnel metal wall (kJ/kg-K)
D	period of steady-state dwell (min)
E	energy (kJ)
GN_2	gaseous nitrogen
h	enthalpy (kJ/kg)
J	joules
K	gain constant or Kelvin
LN_2	liquid nitrogen

LNC	liquid nitrogen consumption
$\dot{m}$	mass flow rate (kg/sec)
mole	mole quantity (0.028016 kg mass of nitrogen)
M	test-section Mach number
N	fan speed (rpm)
P	pressure (atm)
q	flow dynamic pressure (kg/m^2)
R_e	Reynolds number
R	universal gas constant (J/K/mole)
T	gas temperature (K)
t	time or time constant (sec)
u	velocity (m/sec)
U	internal energy (kJ)
V	volume (m^3)
W	mass (kg)
α	cooling capacity of gaseous bleed (kJ/kg)
β	cooling capacity of liquid charge (kJ/kg) $\approx$ (121 + T) kJ/kg
γ	ratio of specific heats
ϕ	test direction parameter
$\cdot$	dot over a quantity refers to its time derivative
Newton	1 kg mass accelerated at 1 m/sec^2
ψ	aeroelastic test direction parameter
Z	compressibility factor (PV/RT)
ρ	density (kg/m^3)
μ	viscosity (Newton-sec/m^2)

Subscripts

a,b,...x	integer indicies
1,2,...m,...n	serial integer indicies
i	integer index
s	static
g	gaseous
F	fan
re	Reynolds number
m	motor
Q'	consumption
low	minimum usable value
sat	saturation
p	pressure

CRYOGENIC TUNNEL PROCESS

A closed-circuit wind tunnel is a device intended for testing scaled models in fluid flow in order to determine their aerodynamic characteristics. The test-section fluid flow over the model is associated with flow parameters to which the aerodynamic characteristics of the model are related. For meaningful scaling of the model data to its full-scale value, it is necessary to maintain the flow similarity between the wind tunnel test-section flow and the full-scale flow. Amongst many flow parameters associated with flow similarity, Mach number and Reynolds number are the most important flow parameters for stationary models. In cryogenic wind tunnels, independent control of the flow parameters can be achieved because the tunnel conditions can be independently controlled (ref. 3).

A closed-circuit cryogenic wind tunnel basically consists of a thermally autonomous pressure vessel designed as an aerodynamically contoured endless duct with a fan whose operation results in a range of flow velocities at the test section. The thermal autonomy of the cryogenic tunnel is realized by thermal insulation to the ambient either external to the metal walls or internal to the metal walls. The operation of the fan results in smooth motion of

the gas around the tunnel circuit accompanied by fan compression and wall friction heating. In order to maintain the tunnel gas temperature in the presence of this heating, liquified form of the test gas, liquid nitrogen, is sprayed into the tunnel. This controlled liquid nitrogen flow allows a cool down or regulation of the tunnel gas temperature. However, in this cooling process, the mass of the tunnel resident gas increases with accompanying increase of pressure. In order to maintain the tunnel pressure at a required value, it is necessary to remove a steady rate of warmer tunnel gas. This is usually achieved by controlled passive discharge of tunnel gas to the atmosphere. This complex interaction of fan-induced heating, liquid nitrogen induced mass increase and enthalpy decrease, and the gas discharge induced mass decrease and enthalpy decrease constitutes the cryogenic tunnel control problem.

This thermodynamic cryogenic tunnel process has been extensively analyzed both graphically and analytically; an explicit, lumped parameter, nonlinear, multivariable model has been synthesized; a real-time, interactive simulator for the cryogenic tunnel has been developed, and finally the model and its real-time simulator have been validated by reconciling the steady-state, quasi-steady and transient responses of the 0.3-m Transonic Cryogenic Tunnel (TCT) to the simulator. This is presented in reference 1.

The developed model has been used to analyze the closed-loop control of the cryogenic tunnel total temperature and total pressure. Further, two nonlinearly gain scheduled pID control laws have been developed with error magnitude dependent control logic to allow large set point imposition and smooth convergence to final set point. The temperature-loop control law has a feed-forward feature to allow rapid Mach number changes. These control laws have been successfully tested on the simulator and have been translated to a microprocessor-compatible flow chart. A microprocessor-based cryogenic tunnel controller has been custom built, and it allows fine tuning of the control process by allowing adjustments of up to 60 tunable coefficients of the control laws. This controller has been successfully commissioned on the 0.3-m TCT to achieve temperature control to ±0.25 K and pressure to ±0.017 atm. The tunnel flow Mach number is directly controlled by the fan speed and can be run on a speed control loop or Mach number control loop. Details of this control

analysis, control law design, flow chart for microprocessor software and the successful operation of the controller are detailed in reference 2.

Thus the problem of precise control of the cryogenic tunnel variables using the various control inputs has been solved. However, the problem of designing the set point set necessary to realize given flow has not been solved. In this report, the design of the tunnel set point set for desired aerodynamic test is addressed.

The wind-tunnel user desires to perform tests on an aerodynamic model so as to obtain the characteristics of the model as a function of the flow parameters: Mach number M and, in the case of a cryogenic tunnel, Reynolds number R_e. The flow dynamic pressure becomes another relevant flow parameter if the tests involve aeroelastic behavior of the model. Otherwise, a minimal flow dynamic pressure q is usually sought.

Translation of the flow parameters M, R_e, and occassionally q to the tunnel set points P, T, and N (or M) constitutes the tunnel test direction design problem. If all the flow parameters R_e, M, and q are a fixed set for a model, there exists a unique P, T, and N for each test. Then, the problem of test direction design is to determine in which order to perform a given set of tests. However, in most tunnel tests, the flow parameter test set R_e and M are fixed with the constraint that the flow dynamic pressure q is to be minimum. Under these conditions, for every R_e-M combination there exists a range of P and T which can provide the necessary flow. However, by constraining the performance of the tunnel to minimum liquid nitrogen consumption or minimum q, an optimal combination of P and T can be determined for a given R_e and M. Under these conditions, the problem of test direction design becomes one of determining the optimal P-T set and then ordering these for minimum energy consumption.

FLOW PARAMETERS

Introduction

The fluid flow in the test section of a wind tunnel can be characterized by many dimensional and nondimensional flow parameters. The parameters of relevance to the present study are Mach number M, Reynolds number R_e, and dynamic pressure q.

Mach Number

The Mach number of a flow in the test section of a wind tunnel is related to the ratio of inertial forces to the elastic forces:

$$M = \frac{u}{a} = \sqrt{\frac{\text{inertial forces}}{\text{elastic forces}}} \tag{1}$$

The velocity of sound in the given a fluid is related to temperature and can be expressed as

$$a = \sqrt{\frac{R\gamma}{\text{mole}} T_s} \tag{2}$$

Reynolds Number

The Reynolds number of a flow in the test section of a wind tunnel is the ratio of the inertial forces and viscous forces occurring in finite length of flow

$$R_e = \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{\rho u \bar{c}}{\mu} \tag{3}$$

The density of nitrogen gas in the test section of a nitrogen tunnel varies as a function of pressure and temperature (ref. 1). As a real gas this is

$$\rho = 338.9 \frac{P_s}{T_s} \left(1 + 250 \frac{P_s}{T_s^2}\right) \qquad \text{kg/m}^3 \tag{4}$$

The velocity of the test gas as a function of gas temperature and flow Mach number is

$$u = 20.38 \sqrt{T_s}\ M \qquad \text{m/sec} \tag{5}$$

where, for nitrogen gas, a mole = 0.028 kg mass, $\gamma = 1.4$, universal gas constant R = 8.31, and hence $\frac{R\gamma}{\text{mole}} = 20.38$.

The viscosity of the test gas, as a function of gas temperature, has been obtained by a simple function fit to Jacobson's data (ref. 5):

$$\mu = 1.082\, T_s^{0.9} \times 10^{-7} \ \text{Newton-sec/m}^2 \tag{6}$$

Hence the flow Reynolds number in a stream of gaseous nitrogen flow after appropriate approximation is

$$R_e = \frac{\rho u \bar{c}}{\mu} = 65650 \frac{P_s}{T_s^{1.4}} \bar{c}\, M \times 10^6 \tag{7}$$

If the tunnel flow is isentropic, the following relationship between total and static quantities in the flow can be used with $\gamma = 1.4$:

$$P_s = P \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-\frac{\gamma}{(\gamma - 1)}} \quad ; \quad T_s = T \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-1} \tag{8}$$

Then

$$R_e = 65650 \frac{P}{T^{1.4}} \bar{c} \frac{M}{\left(1 + 0.2\, M^2\right)^{2.1}} 10^6 \tag{9}$$

Dimensionally, R_e has no dimension:

$$\frac{\rho u \bar{c}}{\mu} = \frac{\dfrac{\text{Newton}}{m^3} \cdot \dfrac{\sec^2}{m} \cdot \dfrac{m}{\sec} \cdot m}{\dfrac{\text{Newton - sec}}{m^2}} \tag{10}$$

For the 0.3-m TCT, which has a typical $\bar{c}$ = 0.1525 m,

$$R_e = 10020 \frac{P}{T^{1.4}} \frac{M}{\left(1 + 0.2 M^2\right)^{2.1}} 10^6 \tag{11}$$

In deriving the expression for Reynolds number of nitrogen flow, perfect gas behavior has been assumed in respect to density. However, since nitrogen gas behaves slightly differently, appropriate corrections for this can be made. Perhaps the best method for introducing this correction is to use the compressibility factor Z. Then

$$R_e = \frac{\rho u \bar{c}}{\mu}$$

$$= \frac{P_s}{ZRT_s} \frac{\sqrt{Z\gamma RT_s}}{\mu} M \bar{c} \tag{12a}$$

$$R_e = \frac{65650}{\sqrt{Z}} \frac{P}{T^{1.4}} \frac{M}{\left(1 + 0.2 M^2\right)^{2.1}} \bar{c} \; 10^6 \tag{12b}$$

However, for purposes of analysis, equation (9) has been used in subsequent treatment, and equation (12b) for final numerical computations as detailed in the Appendix (ref. 3).

Dynamic Pressure

Dynamic pressure is a flow parameter of structural loading significance and corresponds to stream kinetic energy transformed into potential energy as a structural load:

$$q \propto \frac{PM^2\gamma}{2} \quad ; \quad q = \frac{PM^2\gamma}{2}\, 10330 \text{ kg/m}^2 \tag{13}$$

Sensitivity Analysis

The tunnel flow parameters R_e and q are functions of the tunnel variables T and P with Mach number. The following sensitivity analysis provides an understanding of how the flow parameters vary with the tunnel variables:

$$R_e = 65650 \frac{P}{T^{1.4}} \bar{c} \frac{M}{\left(1 + 0.2\ M^2\right)^{2.1}} \tag{14}$$

The sensitivity of R_e to P is

$$\frac{\delta R_e}{R_e} = \frac{\delta p}{P} \tag{15}$$

The sensitivity of R_e to T is

$$\frac{\delta R_e}{R_e} = \left(1 + \frac{\delta T}{T}\right)^{1.4} - 1 \simeq 1.4\ \frac{\delta T}{T} \tag{16}$$

The sensitivity of R_e to M is

$$\frac{\delta R_e}{R_e} \simeq \frac{\delta M}{M} \tag{17}$$

Similarly, the sensitivity of dynamic pressure q to tunnel variables can be determined. The sensitivity of q to P is

$$\frac{\delta q}{q} = \frac{\delta P}{P} \tag{18}$$

The sensitivity of q to M is

$$\frac{\delta q}{q} = \left(1 + \frac{\delta M}{M}\right)^2 - 1 \simeq 2\,\frac{\delta M}{M} \tag{19}$$

These sensitivity expressions can be used to determine the control accuracies and the sensor accuracies based on the desired flow parameter stability.

In figure 1, the allowable error in the control of total temperature in a cryogenic tunnel for the given stability in Reynolds number of 0.5, 1, or 1.5 percent is detailed. Typically the allowable error ranges from 0.35 K up to 3 K.

In figure 2, the allowable uncertainty or error in the control of total pressure in a cryogenic tunnel for a given stability in Reynolds number of 0.5, 1, or 1.5 percent is illustrated. Typically the allowable error ranges from 0.009 atm up to 0.1 atm depending on the tunnel pressure.

It may be noted that the 0.3-m TCT closed-loop controller is able to hold temperature to ±0.25 K and pressure to ±0.017 atm, and hence is generally able to hold Reynolds number to about 0.6 to 0.7 percent.

If the deviations in pressure, temperature, and Mach number occur concurrently, the uncertainty in Reynolds number is much more, and can be estimated as

$$\frac{\delta R_e}{R_e} = \sqrt{\left(1.4\,\frac{\delta T}{T}\right)^2 + \left(\frac{\delta P}{P}\right)^2 + \left(\frac{\delta M}{M}\right)^2} \tag{20}$$

By assuming the contributions of errors in P, T, and M to be weighted equally, for 1 percent uncertainity in R_e, the uncertainities allowed in tunnel conditions are about 0.4 percent in T, 0.6 in P and 0.6 percent in M.

TEST DIRECTION PARAMETER

Introduction

A test direction parameter ϕ is now defined using the expression for Reynolds number in a cryogenic tunnel:

$$R_e = 65650\ \bar{c}\ \frac{P}{T^{1.4}}\ \frac{M}{\left(1 + 0.2\ M^2\right)^{2.1}}\ 10^6 \tag{21}$$

The test direction parameter ϕ is defined as

$$\phi \triangleq \frac{R_e}{M}\left(1 + 0.2\ M^2\right)^{2.1} \triangleq 65650\ \bar{c}\ \frac{P}{T^{1.4}}\ 10^6 \tag{22}$$

The test direction parameter has unique properties which can be exploited in test direction design and economical operation of cryogenic wind tunnels. It can be observed that once the required flow parameters R_e-M are known, ϕ is fixed and the tunnel temperature T and pressure P are uniquely related:

$$\left.\begin{aligned} \phi &= K_{re}\frac{P}{T^{1.4}} \\ P &= \frac{\phi}{K_{re}}\ T^{1.4} \\ T &= \left(\frac{K_{re}}{\phi}\ P\right)^{\frac{1}{1.4}} \end{aligned}\right\} \tag{23}$$

where

$$K_{re} = 65650\ \bar{c} \times 10^6$$

Thus, knowledge of the test direction parameter uniquely describes choices of P and T as described by equation (23). For most cryogenic tunnel tests, the relationship of equation (23) provides the basis for optimal choice of the test direction.

However, in aeroelastic tests involving fixed choice of R_e-M-q, the tests conditions P-T-M are fixed as in

$$P = \frac{2q}{M^2\gamma} \times \frac{1}{10330} \tag{24}$$

and where T is defined as in equation (23).

Properties of Test Direction Parameter

The test direction parameter ϕ uniquely fixes the ratio $P/T^{1.4}$. Using the 0.3-m TCT as an illustrative example, a plot of R_e vs. M for loci of constant ϕ is shown in figure 3. The tunnel flow Mach number varies typically from 0.4 to 1, and the flow Reynolds number from about 0.5×10^6 to 72×10^6. Eleven loci of ϕ varying from 5×10^6 to 105×10^6 in steps of 10×10^6 are shown in figure 3.

Consider a typical constant ϕ locus A_2E_2 corresponding to $\phi = 105 \times 10^6$. This locus represents a range of Reynolds number from about 39×10^6 at $M = 0.4$ to 72×10^6 at $M = 1.0$. The locus A_2E_2 corresponds to fixed $P/T^{1.4}$.

The choice of tunnel total pressure P and temperature T of a cryogenic tunnel is limited to its operating envelope. Figure 4 illustrates the tunnel ideal operating envelope $1 \leq P \leq 6$ atm and $T_{sat} \leq T \leq 350$ K. The pressure envelope limit is imposed by the maximum safe pressure based on appropriate pressure vessel code and by the fact that for passive gas bleed the tunnel must function at or above atmospheric pressure. The low temperature limit is imposed by the need to keep the tunnel flow in gaseous phase. The upper limit is dictated by structural considerations.

Figure 4 provides the loci of constant test direction parameter ϕ in the plane P vs. T. Eleven loci of $\phi = 5 \times 10^6$ to 105×10^6 in steps of 10×10^6 cover the tunnel envelope. Each locus corresponds to the choice of P-T combination for a given R_e-M and hence ϕ. Consider a typical locus DE corresponding to $\phi = 15 \times 10^6$. A choice of P from 1 to 6 atm and T from 115 to 350 K can be made along line DE. Further, ϕ corresponds to a range of R_e from 5.6×10^6 to 10.2×10^6 and of M from 0.4 to 1.0. Thus figure 4 provides an indication of the flexibility involved in translating R_e-M to P-T.

Figure 4 further illustrates the limitations imposed by the tunnel fan power and fan speed derived using the following identities from reference 2:

$$\text{Power limit} = 1900 \text{ kW} = \frac{K_F P \sqrt{T}\, M^3}{\left(1 + 0.2\, M^2\right)^3} \tag{25}$$

$$\text{Fan speed limit} = 5600 \text{ rpm} = 597\, M \sqrt{T}\, (1 - 0.3\, M)\, P^{-0.035} \tag{26}$$

The power and speed boundaries limit the choice of P and T for a given M and ϕ.

It is evident from figure 4 that, at a given ϕ, the tunnel total pressure increases along the locus DE and hence the flow dynamic pressure q also increases along the locus DE. Although the minimum ideal tunnel total pressure boundary is 1 atm and the minimum ideal total temperature boundary is T_{sat}, the minimum usable total pressure in passive bleed mode is about 1.2 atm and the minimum usable total temperature is a T_{low}, which exceeds T_{sat} by a safe margin. The choice of lowest usable tunnel pressure for a given ϕ occurs along the locus AB_1B of figure 4, and either along the T_{sat} locus BC or along the safe temperature locus T_{low} corresponding to B_1C_1. Hence the ideal locus ABC or the safe usable locus AB_1C_1 turns out to be the minimum dynamic pressure locus for any set of test ϕ. As will be demonstrated later, the locus ABC/AB_1C_1 also turns out to be the minimum liquid nitrogen consumption locus for a given set of test ϕ. Thus figures 3 and 4 provide an insight into the properties of test direction parameter ϕ in respect to R_e-M and P-T as well as to q.

MINIMUM ENERGY TEST DIRECTION DESIGN

Introduction

Consider a model test program in a cryogenic wind tunnel wherein aerodynamic data on the model is desired at a number of flow parameter conditions of R_e-M combinations. Depending upon the number of angles of attitude, a minimum dwell time of steady flow is associated with each flow condition. Let the flow parameter condition set be expressed as $\left(R_e\text{-M-D}\right)_a$, $\left(R_e\text{-M-D}\right)_b$, ...$\left(R_e,\text{M},\text{D}\right)_x$ where subscripts a,b,...x correspond to arbitrarily ordered n conditions of test, and D corresponds to dwell time.

The problem of test direction design in a cryogenic wind tunnel is, firstly, the translation of the flow parameter test condition $\left(R_e\text{-M-D}\right)_i$ to the tunnel variable condition $\left(\text{P-T-M-D}\right)_i$ and, secondly, ordering the integer set i = a,b,...x to an execution order i = 1,2,3,...n such that overall performance is optimal either in terms of minimum liquid nitrogen or least dynamic pressure or both.

The problem of determining the minimum liquid nitrogen consumption strategy in realizing a given test program in R_e-M is now considered in two cases. In the first case, the liquid nitrogen consumption in cooling the tunnel shell is ignored and the problem of choosing the best P-T combination for least liquid nitrogen consumption is addressed. In the second case, the total liquid nitrogen consumption in cooling the tunnel, in buildup of tunnel pressure and in the best choice of P-T for given R_e-M such that least consumption occurs is detailed.

Case 1: Minimum Liquid Nitrogen Consumption Strategy, Considering Fan Heat Only, for a Given Test Program in R_e-M

The fan energy released into the tunnel gas stream when the tunnel conditions are pressure P, temperature T and Mach number M is

$$E = \frac{K_F\, P \sqrt{T}\, M^3}{\left(1 + 0.2\, M^2\right)^3}\, 60\, D \qquad (27)$$

The liquid nitrogen consumed in maintaining the tunnel temperature at T is (ref. 1),

$$LCN = \frac{E}{\beta} = \frac{60\ D\ K_F\ P \sqrt{T}\ M^3}{\left(1 + 0.2\ M^2\right)^3 \left(121 + T\right)} \quad kg \tag{28}$$

In a given test program, with the desired set of R_e-M known, the test direction parameter ϕ and hence $\frac{P}{T^{1.4}}$ is fixed. As described by equation (23):

$$T = \left(\frac{K_{re}}{\phi} P\right)^{\frac{1}{1.4}} \quad ; \quad P = \frac{\phi}{K_{re}} T^{1.4}$$

where

$$K_{re} = 65650\ \bar{c} \times 10^6$$

Substituting this expression in equation (28), the liquid nitrogen consumption LCN over a period of 60 D seconds of steady flow can be expressed as a function of temperature only:

$$LCN = K_Q \frac{T^{1.9}}{121 + T} \tag{29}$$

where

$$K_Q = \frac{60\ D\ \phi\ M^3}{\left(1 + 0.2\ M^2\right)^3 K_{re}}$$

The liquid nitrogen consumption can be minimized as a function of T:

$$\frac{\partial(LCN)}{\partial(T)} = K_Q\ T^{0.9} \left[\frac{1.9\ (121 + T) - T}{\left(121 + T\right)^2}\right] = 0 \tag{30}$$

Hence extrema occur at T = -255 K or T = 0. Obviously these extrema do not occur in the physically realizable range of $T_{sat} \leq T \leq 350$ K. The lowest value of LCN occurs at the lowest physically realizable value of $T = T_{sat}$ as evident from inspection of equation (29).

However, in operating a cryogenic tunnel, the stream local static temperature over the model is far less than the stream total temperature T. Hence a safe margin is necessary in choice of T_{low} such that $T_{low} = T_{sat}$ + margin. The magnitude of margin is a function of flow Mach number. In reference 6, determination of safe margin based on theoretical and experimental analysis is detailed. For purposes of this report, a safe maximum margin of 12 K has been used for all computations. The saturation vapor phase boundary of nitrogen can be expressed, on the basis of simple function fit for data from reference 5, as

$$T_{sat} = 50 + 27.34\, P^{0.296} \text{ kelvin} \tag{31}$$

The locus of vapor phase boundary T_{sat} is shown in figure 4 as locus BC. The liquid consumption can also be expressed as a function of pressure P alone as

$$\text{LCN} = K_p \frac{P^{1.36}}{K_1 + P^{0.71}} \tag{32}$$

This function can be minimized for P:

$$\frac{\partial(\text{LCN})}{\partial(P)} = \frac{\left\{1.36 \left[K_1 P^{0.36} + P^{1.07}\right] - 0.71\, P^{1.07}\right\}}{\left(K_1 + P^{0.71}\right)^2} \tag{33}$$

It can be observed that again the minima occur at either P = 0 or $P = -f_1(K_1)$. Both these values of P are physically unrealizable since they fall out of the tunnel range $P_{low} \leq P \leq 6$ atm. Further, it can be noted that lowest LCN occurs at P_{low} in the physical world as can be observed from equation (32).

Hence, given a set of R_e-M combinations to be realized, the best operating P-T combination occurs either along the locus P_{low}-T or T_{low}-P locus. The least liquid nitrogen locus AB_1C_1, wherein AB_1 is the P_{low}-T segment and B_1C_1 is the T_{low}-P segment, is shown in figure 4. Obviously, given the desired R_e-M combination, P_{low} is assumed and T determined as per equation (23). The point A corresponds to the lowest ϕ and point C corresponds to highest ϕ. In the 0.3-m TCT, segment AB corresponds to ϕ from 1×10^6 to 27.5×10^6. Segment BC corresponds to ϕ from 27.5×10^6 to 105×10^6. However, the usable locus B_1C_1 corresponds to ϕ from 24×10^6 to 87×10^6.

The problem of ordering the flow parameter test set $\left(R_e\text{-M-D}\right)_a$, $\left(R_e\text{-M-D}\right)_b$, ... $\left(R_e\text{-M-D}\right)_x$ is now considered. Each operating flow parameter point is associated with a finite ϕ, and hence the test program set has a test direction parameter set of $\phi_a, \phi_b, \ldots \phi_x$. The numerical values of $\phi_a, \phi_b, \ldots \phi_x$ are determined, and these are reordered such that $\phi_1 \leq \phi_2 \leq \ldots \phi_n$. The test direction providing least liquid consumption is $\phi_1, \phi_2, \ldots \phi_n$.

Consider the first test point ϕ_1, corresponding to $\left(R_e\text{-M-D}\right)_1$ belonging to the set $\phi_a, \ldots \phi_x$. This test direction parameter ϕ_1 is now translated into the tunnel operating condition P_1, T_1, M_1, D_1. Since the value of ϕ_1 is the lowest, as shown in figure 4, the least energy operating point is likely to be along the locus AB_1. The choice of pressure is $P_1 = P_{low} = 1.2$ atm, and the complementing temperature T_1 is determined from equation (23), in the range $T_{low} \leq T_1 \leq 350$ K. After performing tests at ϕ_1 for period D_1 the tunnel is moved to ϕ_2. Note that $\phi_2 \geq \phi_1$.

The new tunnel pressure P_2 and temperature T_2 are estimated from equation (23), at the low tunnel pressure of $P_2 = P_1 = P_{low}$. Obviously, temperature $T_2 \leq T_1$ in the range $T_{low} < T_2 < 350$ K. In reaching T_2, cooling of the tunnel shell from T_1 to T_2 is involved. This process is carried out for $\phi_3, \ldots$ up to a ϕ_i wherein $T_i \leq T_{low}$. When this stage is reached the new tunnel conditions are estimated on the basis of $T_i = T_{low}$, and P_i is estimated from equation (23). Subsequently, ϕ_{i+1} to ϕ_n are translated to $P_{i+1} - T_{i+1}$, etc. and are likely to occur along locus B_1C_1 of figure 4.

In this process of translation of $[\phi]$ to [P-T], it is obvious that, since $\phi_1 \leq \phi_2 \leq \ldots \phi_n$ the tunnel conditions also bear the relation $P_1 \geq P_2 \geq P_3 \ldots \geq P_n$ and $T_1 \leq T_2 \leq \ldots$ along locus AB_1. However, the tunnel temperature bears

the relation $T_n \geq T_{n-1} \geq \ldots$ along locus B_1C_1, since the vapor phase boundary has positive slope in the P-T plane. Thus the execution order involves no reversals in tunnel conditions and proceeds monotonically from near ambient P-T to tunnel extremes, thereby assuring minimal consumption of liquid nitrogen.

Case 2: Minimum Liquid Nitrogen Consumption Strategy, Considering Fan Compression, Metal Shell Cooling, and Mass Buildup, for a Given Test Program in R_e-M

Consider a cryogenic wind tunnel inactive at ambient conditions. To use this tunnel to realize a tunnel flow of R_e-M for a period of 60 D seconds and then terminate the test, a finite amount of liquid nitrogen consumption is involved. The problem of minimizing this total liquid nitrogen consumption as a function of temperature is now considered.

The energy required to cool the tunnel from ambient of say 300 K to a temperature T is dominantly controlled by metal enthalpy between the 2 temperatures. This has been experimentally verified for the 0.3-m TCT (ref. 1). To an acceptable degree of accuracy, the liquid nitrogen required to cool the tunnel from 300 K to T, at any pressure is

$$\frac{W_t C_m + W_g C_v}{\beta} (300 - T) \quad \text{kg} \tag{34}$$

It is assumed that tunnel cooling occurs at lowest pressure of 1.2 atm and a low convection Mach number of M = 0.3 or less. In estimating the mass of gas W_g, a pressure of P can be used at an average temperature of $\frac{300 + T}{2}$.

The liquid nitrogen consumption in building the tunnel pressure can be estimated from the density expression at constant temperature as

$$341 \frac{P - 1}{T} V \quad \text{kg} \tag{35}$$

where the pressure buildup is from 1 atm to P.

The total liquid nitrogen consumption in performing a cryogenic tunnel test at R_e-M for 60 D seconds is

$$LCN = \frac{60\ D\ K_F\ P\sqrt{T}\ M^3}{\left(1 + 0.2\ M^2\right)^3\ (121 + T)} + \frac{W_g C_v + W_t C_m}{(121 + T)}\ (300 - T) + \frac{341\ V}{T}\ (P - 1) \tag{36}$$

Since the test direction parameter ϕ is known from equation (23) as

$$P = \frac{\phi}{K_{re}}\ T^{1.4}$$

Hence

$$LCN = K_Q\ \frac{T^{1.9}}{(121 + T)} + \frac{W_t C_m + W_g C_v}{(121 + T)}\ (300 - T) + \frac{341\ V}{T}\left(\frac{\phi}{K_{re}}\ T^{1.4} - 1\right) \tag{37}$$

The liquid nitrogen consumption consists of three terms. As temperature T moves down from 300 K, the second term grows linearly. At its maximum, when $T = T_{low}$ the second term has a maximum of about 1800 kg. The third term grows linearly with pressure, and, at the maximum tunnel pressure of 6 atm, the liquid nitrogen involved corresponds to about 350 kg. The first term is power and dwell period D dependent.

It can be noted that cooling the tunnel involves nearly 1800 kg of liquid which can be eliminated if the tunnel run can be performed at high temperatures. The minimization of equation (36) analytically with respect to T would yield this solution. In the present analysis, a numerical estimation of LCN has been carried out as a function of temperature and the results are presented in figures 5 to 14. These correspond to R_e of 10×10^6 to 50×10^6 in steps of 10×10^6 at M = 0.6 and M = 0.9. The program generates combinations of P and T necessary to realize given R_e-M, while conforming to physical realizability.

Figure 5 is a plot of LNC as a function of T in the pressure range of $1.2 \leq P \leq 6$ for various dwell times ranging from 2 to 22 min. When the dwell period is 2 min, the test program for realizing $R_e = 10 \times 10^6$ at $M = 0.6$ is most economical only at 300 K and not 105 K. However, as the dwell time increases, the loci of LCN which have a negative slope at $D = 2$ gradually turn positive. At a 22-min dwell time, the locus has a positive slope, indicating that operation of the tunnel at 105 K is most economical. Obviously the metal enthalpy term of 1800 kg is more than saved at high dwell times and hence makes cool down worthwhile.

In figure 6, $R_e = 20 \times 10^6$ at $M = 0.6$, the range of usable temperature is reduced. The issue here is whether to cool the tunnel to 200 K or down to 85 K. Again the loci of LCN provide the answer in that for low dwells full cool down is not necessary, whereas, for large dwell periods in excess of 10 min, it is necessary to cool the tunnel down to save liquid nitrogen.

The progressive reduction of temperature range for $R_e = 30 \times 10^6$ to 50×10^6 at $M = 0.6$ can be noted in figures 7 to 9. In figure 9, the choice of temperature is only a few degrees.

Similar results are illustrated in figures 10 to 14 for $R_e = 10 \times 10^6$ to 50×10^6 at $M = 0.9$. Again, similar conclusions regarding dwell time can be observed. In summary, as long as the cryogenic tunnel experimental dwell time is of considerable length, involving a reduction in liquid consumption by cool down in excess of 1800 kg, the least energy consumption locus is the minimum temperature/minimum pressure locus. However, for very short experiments there exists another choice of operation at highest feasible temperature, with the associated higher q flows. Major cryogenic tunnels are not likely to run for such short periods, and so the second optimal solution is meaningless.

LOCUS OF OPTIMAL TEST DIRECTION

In the previous sections, the problem of test direction design in a cryogenic wind tunnel was defined as one, firstly, of ordering the desired flow parameter conditions $(R_e\text{-M-D})_a, \ldots (R_e\text{-M-D})_x$ as $(R_e\text{-M-D})_1, \ldots (R_e\text{-M-D})_n$

and, secondly, of translating these operating flow parameters into tunnel conditions $(P\text{-}T\text{-}M\text{-}D)_1, \ldots (P,T,M,D)_n$. In doing so the constraint of minimizing liquid nitrogen consumption was fulfilled. In addition, the chosen flow dynamic pressure was the minimum ever achievable.

The best tunnel operating locus in the tunnel pressure-temperature envelope turned out to be along the minimum usable tunnel pressure line at varying temperature in the range $T_{low} \leq T \leq 350$ K and secondly along the minimum usable temperature line at varying pressure in the range $P_{low} \leq P \leq 6$ atm. The locus AB_1C_1 of figure 15 illustrates the best operating locus. Surprisingly, the inference that can be drawn by this study is that a cryogenic tunnel need not operate at all at high pressures when at any temperature other than T_{low}. The locus AB_1C_1 is capable of providing the total performance capability of the cyrogenic tunnel in R_e-M. Need for working at any other point in the P-T plane of figure 4 or 15 arises only if R_e-M-q combinations are desired such that q is a dynamic pressure higher than the minimum along line AB_1C_1.

The best test direction locus ABC is transformed into the energy state plane U vs. W_g and is shown in figure 16. The energy state diagram shows the internal energy-mass plane with loci of constant pressure, constant temperature, and constant test direction parameter ϕ. The minimum liquid nitrogen consumption test direction locus AB_1C_1 moves, firstly, along the minimum tunnel pressure locus AB_1 and then along the minimum temperature locus B_1C_1.

The test direction design problem can be exemplified by points A_2, B_2, C_2, and D_2 of figure 3. Consider four Reynolds number-Mach number flows of $R_e = 39 \times 10^6$ - $M = 0.4$ at point A_2, $R_e = 42 \times 10^6$ - $M = 0.58$ at point B_2, $R_e = 48 \times 10^6$ - $M = 0.57$ at point C_2, and $R_e = 51 \times 10^6$ - $M = 1.0$ at point D_2. The normal tendency for a tunnel user would be to perform tests in order $A_2B_2C_2D_2$. However the estimation of ϕ for each case shows that ϕ is 105×10^6, 85×10^6, 95×10^6, and 75×10^6, respectively, for the points A_2, B_2, C_2, and D_2. The preferred order of execution as analyzed previously turns out to be $D_2B_2C_2A_2$ in ascending order of ϕ and would involve tests during a pressure buildup at T_{low}.

The problem of test direction design in a test program wherein the desired conditions are $(R_e\text{-M-D-q})_a$, $(R_e\text{-M-D-q})_b$,... $(R_e\text{-M-D-q})_n$ can now be considered. This type of test would probably be an aeroelastic test. Since all the three flow parameters are given, there is no minimum liquid consumption problem. However, the problem of ordering does exist theoretically. Given $(R_e\text{-M-D-q})$, the choice of P-T is unique, since $q = \frac{PM^2\gamma}{2}$ 10330

$$P = \frac{2q}{M^2\gamma} \times \frac{1}{10330} \tag{38a}$$

Then

$$T = \left[\frac{R_e\left(1 + 0.2\ M^2\right)^{2.1}}{65650\ \bar{c}\ M}\right]^{\frac{1}{1.4}} \tag{38b}$$

The problem of ordering could be conceived through another test parameter ψ:

$$\psi \triangleq \frac{R_e M\left(1 + 0.2\ M^2\right)^{2.1}}{q} \triangleq \frac{9.07}{T^{1.4}}\ 10^6 \tag{39}$$

This expression can be easily derived from equations (9) and (24).

The dynamic pressure ranges from 1,168 kg/m^2 at P = 1 atm and M = 0.4 to 43,400 kg/m^2 at P = 6 atm and M = 1.0. The test direction for aeroelastic tests can be ordered in ψ. However, since ψ is directly proportional to $T^{-1.4}$, the aeroelastic tests can be planned monotonically without having to conceive of or evaluate ψ. Hence, for aeroelastic tests, equations (38a) and (38b) are used to evaluate P and T, and then the tests are directly ordered in T. Thus $(R_e\text{-M-D-q})_a$,... $(R_e\text{-M-D-q})_x$ is transformed to $(\text{P-T-M-D})_a$,... $(\text{P-T-M-D})_x$. The temperature values T_a,...T_x are then used to reorder them such that $T_1 \geq T_2 \geq \ldots T_n$ where 1,2,...n are reordered set a,b,...x.

TYPICAL CRYOGENIC TUNNEL TEST DIRECTION DESIGN

Introduction

A computer program in Fortran which provides the typical test direction design for a given test program consisting of desired Reynolds number-Mach number flow conditions is detailed in the Appendix. The program is a simple mechanization of the test direction design procedure analyzed in previous sections. The program uses fixed tunnel parameters specific to the 0.3-m TCT viz volume of 14.1 m^3, metal structure weight of 3200 kg, fan power constant, and the cooling capacity of liquid nitrogen.

The program initially reads in the number of test points n and associated flow parameter set R_e-M-D. The test direction parameter ϕ is determined for each of the flow parameters, and the test flow parameter set is reordered such that $\phi_1 \leq \phi_2 \leq \ldots \phi_n$. Secondly, the program translates each flow parameter condition R_e-M-D into tunnel test condition P-T-M-D such that minimum liquid nitrogen is consumed in the process. The estimate of liquid nitrogen required for fan power cancellation and for transition of the tunnel from the previous condition to the present condition is estimated. It is assumed that the metal shell cooling occurs at low Mach number and low pressure, thereby assuring minimum wastage of liquid. The computer program incorporates real gas correction in the form of compressibility effects.

Example

A specific test program realizing 38 flow parameter combinations ranging from $R_e = 0.9 \times 10^6$ at $M = 0.1$ to $R_e = 50 \times 10^6$ at $M = 0.8$ is now considered. The table in figure 17 describes the 38 flow parameter conditions with the estimated dwell time.

The problem is to decide the order in which these flow parameter conditions are to be realized in the cryogenic tunnel and to translate the flow parameters to tunnel conditions P-T which provide minimum consumption of liquid nitrogen.

The program is used to evaluate the test direction parameter ϕ in the order $\phi_1 \leq \phi_2 \leq \ldots \phi_{38}$. The result of this process is shown in figure 18. The tests in the tunnel are to be performed in the same order. Note that the

lowest $R_e = 0.9 \times 10^6$ at M = 0.1 is listed as the twelfth test point and the highest $R_e = 50 \times 10^6$ at M = 0.84 as the thirty-first test point.

The Fortran program has been used to translate the flow parameter test conditions of figure 18 to the tunnel operating conditions P and T. The program further estimates the liquid nitrogen consumed to cancel fan power and liquid nitrogen consumed for tunnel condition transition. The tolerances on the tunnel pressure and temperature control to assure half percent uncertainty in the flow Reynolds number are also evaluated. The table of figure 19 thus provides the solution to the problem of test direction design for conditions tabulated in figure 17.

It may be noted that the first test point starts at the lowest tunnel pressure of about 1.2 atm and the highest tunnel temperature of 235 K. The subsequent operating points occur in the cool down direction. When the test ϕ reaches 24×10^6, corresponding to $R_e = 15 \times 10^6$ at M = 0.8 the temperature limit is encountered. Subsequent test points occur at the low temperature limit of T_{sat} + margin, but at increasing pressures. It is obvious that the tunnel conditions move monotonically through the test range, assuring minimal transition in liquid nitrogen consumption.

The test program can be run in the reverse order also without any extra loss of liquid nitrogen. The program finally prints the total liquid nitrogen consumption estimate to perform the desired test.

CONCLUDING REMARKS

The cryogenic wind tunnel is a device intended for generating very high Reynolds number flows past aerodynamic models. This increase of flow Reynolds number is obtained by cooling the test gas, nitrogen, by use of liquid nitrogen sprayed into the tunnel.

The cost of using a cryogenic tunnel is dominated by the cost of liquid nitrogen. Typically the cost of energy in the form of liquid nitrogen is nearly 80 to 100 times the cost of equivalent electrical energy. Thus the liquid nitrogen consumption in a cryogenic tunnel test program should be minimal.

An important requirement of the cryogenic tunnel use is that the user should conceive of the wind tunnel test program on the basis of desired flow R_e, flow M, and dwell period. The dwell period depends upon the number of attitudes of the model and data scan time. In the contemporary, ambient temperature tunnel usage practice, the tunnel users usually conceive of the test on the basis of Mach number and tunnel blowing pressure.

In this report, analysis of techniques for ordering a test program and translating the tunnel flow parameters into the tunnel flow conditions P and T has been carried out. A test direction parameter ϕ has been defined to provide the basis for ordering the test program. Secondly, a simple extremum determination analysis has been carried out to determine the best tunnel conditions P-T involving least energy consumption. This problem has been shown to be valid only when the flow dynamic pressure q is unconstrained. Further, the test liquid nitrogen consumption locus has been shown to be the least dynamic pressure locus also.

In the case of aeroelastic tests involving R_e-M-q, there exists a single P-T-M set for each flow condition. The test order here is based on tunnel temperature T being chosen monotonically.

In the procedure used for translating the flow parameters to tunnel variables, a single real gas correction for compressibility has been made. Better accuracy can be obtained in translating R_e-M to P-T by incorporating viscosity effects. However, this does not affect the test direction.

In conclusion, the problem of test direction design in cryogenic wind tunnels has been defined, analyzed and basically resolved in this report.

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APPENDIX

PROGRAM LISTING FOR TEST DIRECTION DESIGN

```
      PROGRAM MAIN(INPUT,OUTPUT,TAPE1,TAPE5=INPUT,TAPE6=OUTPUT)
      DIMENSION M(100),MC(100),U(100),RE(100),D(100),R(100),UR(100),
     1P(100),T(100),DUM(100),DEP(100),DET(100)
CCC PROGRAM VARIABLES
C    INPUT FLOW PARAMETERS
C    RE =REYNOLDS NUMBER * 10E-6
C    M = MACH NUMBER
C    U = TEST DIRECTION PARAMETER * 10E-6
C    D = DWELL,MINUTES
C    EXECUTION ORDER FLOW PARAMETERS
C    R = REYNOLDS NUMBER * 10E-6
C    MC = MACH NUMBER
C    UR = TEST DIRECTION PARAMETER * 10E-6
C    DUM = DWELL,MINUTES
      REAL M,MC
      N=38
CCC  ARG= MARGIN IN TEMPERATURE  TO ONSET OF CONDENSATION
      ARG=12
CCC N = NUMBER FLOW PARAMETER TEST POINTS IN PROGRAM
      G=2.1
CCC READ  INPUT  DATA- FLOW PARAMETER SETS
      READ 4,(RE(I),M(I),D(I),I=1,N)
    4 FORMAT(3F10.2)
      PRINT 16
   16 FORMAT(1H1,//12X*RE*13X*M*13X*D*)
      PRINT 17,(RE(I),M(I),D(I),I=1,N)
   17 FORMAT(3F14.2)
CCC DESIGN  TEST  ORDER  BY  MONOTONIC  ORDERING  OF  TEST  DIRECTION
CC   PARAMETER     U= R/M*((1+0.2*M*M)**2.1)
      DO 5 J=1,N
    5 U(J)=RE(J)*((1+0.2*M(J)*M(J))**G)/M(J)
      DO 6 J=1,N
      UU=-0.001
      DO 10 I=1,N
      X=U(I)-UU
      IF(X.GT.0) GO TO 15
      GO TO 10
   15 UU=U(I)
      L=I
   10 CONTINUE
      NN=N-J+1
      R(NN)=RE(L)
```

```
      MC(NN)=M(L)
      UR(NN)=U(L)
      DUM(NN)=D(L)
      U(L)=-0.001
    6 CONTINUE
      PRINT 19
   19 FORMAT(1H1,//8X*DIRECTION*8X*RE*13X*M*)
      PRINT 20,(UR(I),R(I),MC(I),I=1,N)
   20 FORMAT(3F14.2)
      PRINT 22
   22 FORMAT(1H1,//19X*MARGIN TO ONSET OF CONDENSATION,K*)
      PRINT 23,ARG
   23 FORMAT(31X,F4.1)
      PRINT 21
   21 FORMAT(//5X*RE*9X*M*7X*P*8X*T*6X*DWELL*4X*FANLN*5X*TLN*
     14X*DEP*3X*DET*)
CCC TRANSLATE  RE-M  IN TO  TUNNEL  CONDITIONS  P-T
      SUM=0
      PPP=1
      TTT=300
      DO 100 I=1,N
CCC START  AT  PRESSURE  OF  PLOW = 1.2 FOR  0.3-M TCT
      P(I)=1.2
   25 CONTINUE
      TT=(P(I)*10020*MC(I)/R(I)/((1+0.2*MC(I)*MC(I))**G))**(1/1.4)
CCC  ADJUST FOR COMPRESSIBILITY FACTOR
   26 CONTINUE
      DD=1.37-8.773E-2*TT+4.703E-4*TT*TT-1.386E-6*(TT**3)+
     11.462E-9*(TT**4)
      EE=5.521-1.986E-1*TT+7.817E-4*TT*TT-1.258E-6*(TT**3)+
     15.333E-10*(TT**4)
      ZZ=1-EXP(DD)*P(I)-EXP(EE)*(P(I)**2)
      IF(ZZ.LE.0.5)GO TO 31
      GO TO 32
   31 ZZ=0.5
   32 RR=10020*P(I)/(TT**1.4)*MC(I)/((1+0.2*MC(I)*MC(I))**2.1)/SQRT(ZZ)
CCC   ITERATE FOR COMPRESSIBILITY ADJUSTMENTS TO REYNOLDS NUMBER
      DR=ABS((R(I)-RR)/R(I))
      IF(DR.LE.0.002)GO TO 27
      IF(R(I).GT.RR)GO TO 28
      IF(R(I).LT.RR)GO TO 29
   28 TT=TT-0.01
```

```
      GO TO 26
   29 TT=TT+0.01
      GO TO 26
   27 CONTINUE
CCC CHECK FOR PHYSICAL REALISABILITY OF TEMPERATURE
CCC   CHECK FOR ONSET OF CONDENSATION
CCC    VAPOUR PHASE BOUNDARY IS    50+27.34*(P**0.296)
      X=TT-50-27.34*(P(I)**0.296)-ARG
      IF(X.LT.0) GO TO 30
      GO TO 40
   30 P(I)=P(I)+0.01
      GO TO 25
   40 T(I)=TT
      R1=R(I)
      R2=R(I)*1.005
      P1=R1*(T(I)**1.4)*((1+0.2*MC(I)*MC(I))**G)/10020/MC(I)
      P2=R2*(T(I)**1.4)*((1+0.2*MC(I)*MC(I))**G)/10020/MC(I)
      DEP(I)=(P1-P2)*14.696
      T1=(P1*10020*MC(I)/R1/((1+0.2*MC(I)*MC(I))**G))**(1/1.4)
      T2=(P1*10020*MC(I)/R2/((1+0.2*MC(I)*MC(I))**G))**(1/1.4)
      DET(I)=T2-T1
CCC CHECK FOR PHYSICAL REALISABILITY OF PRESSURE  1.2<P<6
      Y=6-P(I)
      IF(Y.LT.0) GO TO 45
      GO TO 47
   45 CONTINUE
      GO TO 100
CCC ESTIMATE  LN2 REQUIRED FOR FAN COMPRESSION HEAT CANCELLATION
   47 H=57.2*P(I)*SQRT(T(I))*(MC(I)**3)/(121+T(I))
      H=H/((1+0.2*MC(I)*MC(I))**3)
      S=H*DUM(I)*60
      XX=TTT-T(I)
      IF(XX.LT.0)GO TO 43
      GO TO 44
   43 TTT=T(I)
   44 CONTINUE
CCC ESTIMATE LN2 REQUIRED FOR  TUNNEL SHELL COOLING  & PRESSURISATION
      S1=(TTT-T(I))*9+(P(I)-PPP)*4779/T(I)
      SUM=SUM+S+S1
      PPP=P(I)
      TTT=T(I)
CCC PRINT EXECUTION ORDER,TUNNEL STATES,DWELL TIME,ESTIMATED LN2 CONSUMPTION
```

```
C    CONTROL ERROR TOLERANCES ETC
CCC DET = TOLERANCE IN TEMPERATURE, K FOR 1/2 PERCENT STABILITY IN RE
CCC DEP = TOLERANCE IN PRESSURE,PSI FOR  1/2 PERCENT  RE  STABILITY
C    TRAN LN2 = LN2 USED FOR SHELL COOL DOWN & PRESSURISATION
   48 PRINT 50,(R(I),MC(I),P(I),T(I),DUM(I),S,S1,DEP(I),DET(I))
   50 FORMAT(4F9.2,3F9.1,2F6.2)
  100 CONTINUE
      PRINT 59
   59 FORMAT(//2X*TOTAL LN2 USED*)
      PRINT 60,SUM
   60 FORMAT(F12.3)
      STOP
      END
```

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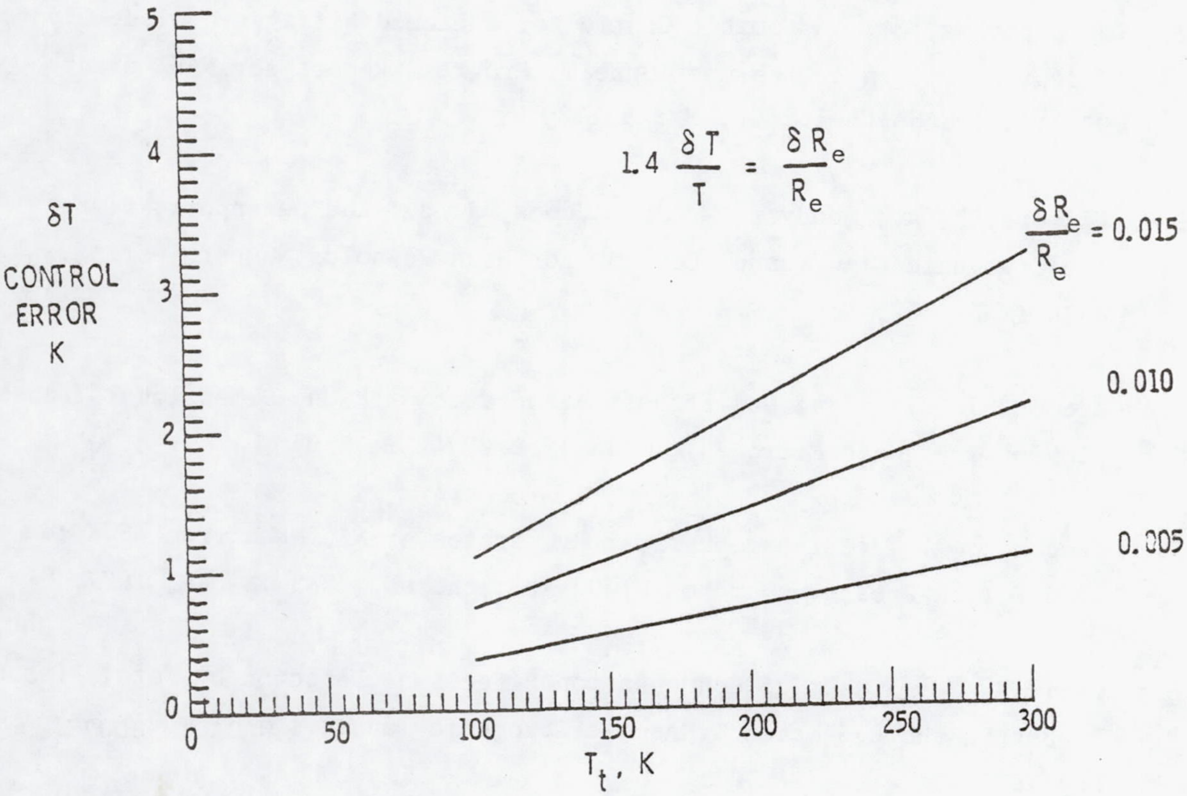

Figure 1. Allowable error in control of tunnel total temperature for a given stability in Reynolds number in a cryogenic wind tunnel.

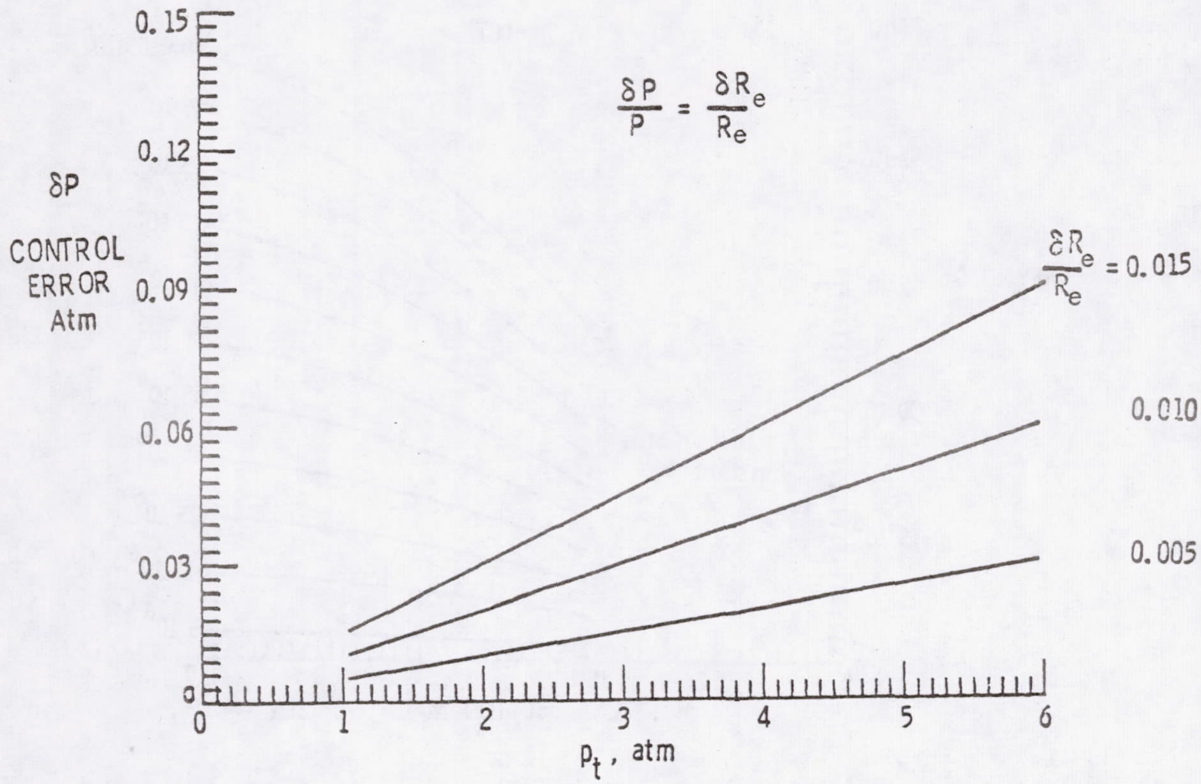

Figure 2. Allowable error in control of tunnel total pressure for a given stability in Reynolds number in a cryogenic wind tunnel.

$$\text{Test direction parameter} = \frac{R_e}{M}(1 + 0.2\,M^2)^{2.1} = 69650\ \frac{P}{T^{1.4}}\ \overline{C}\ 10^6$$

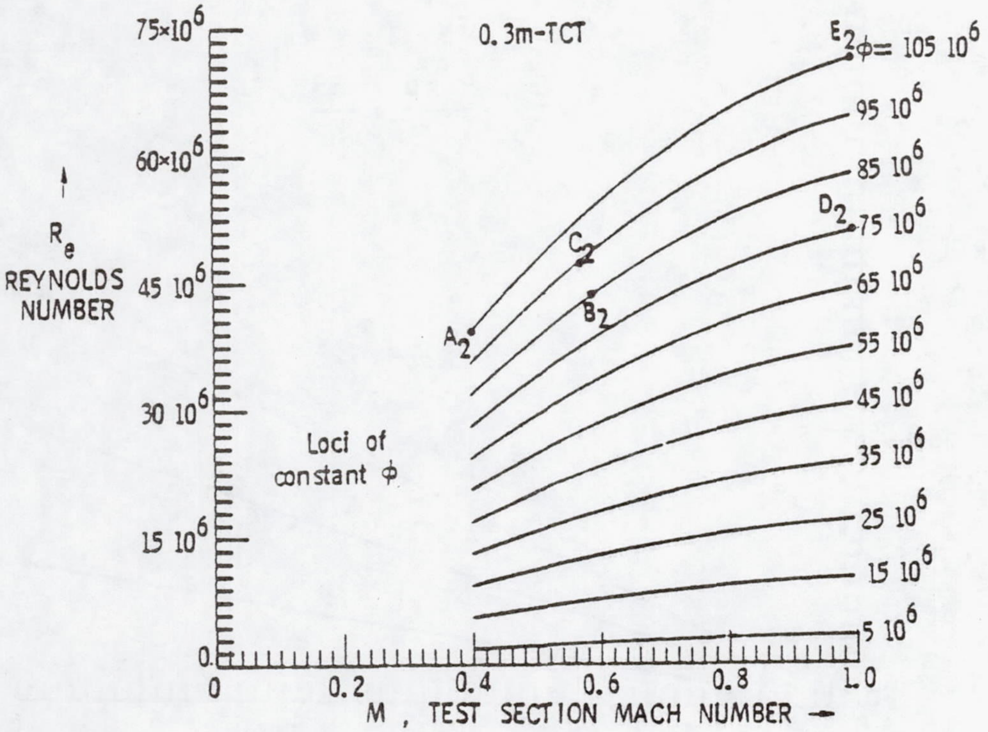

Figure 3. Test direction design in cryogenic wind tunnels—test direction parameter ϕ as a function of R_e and M.

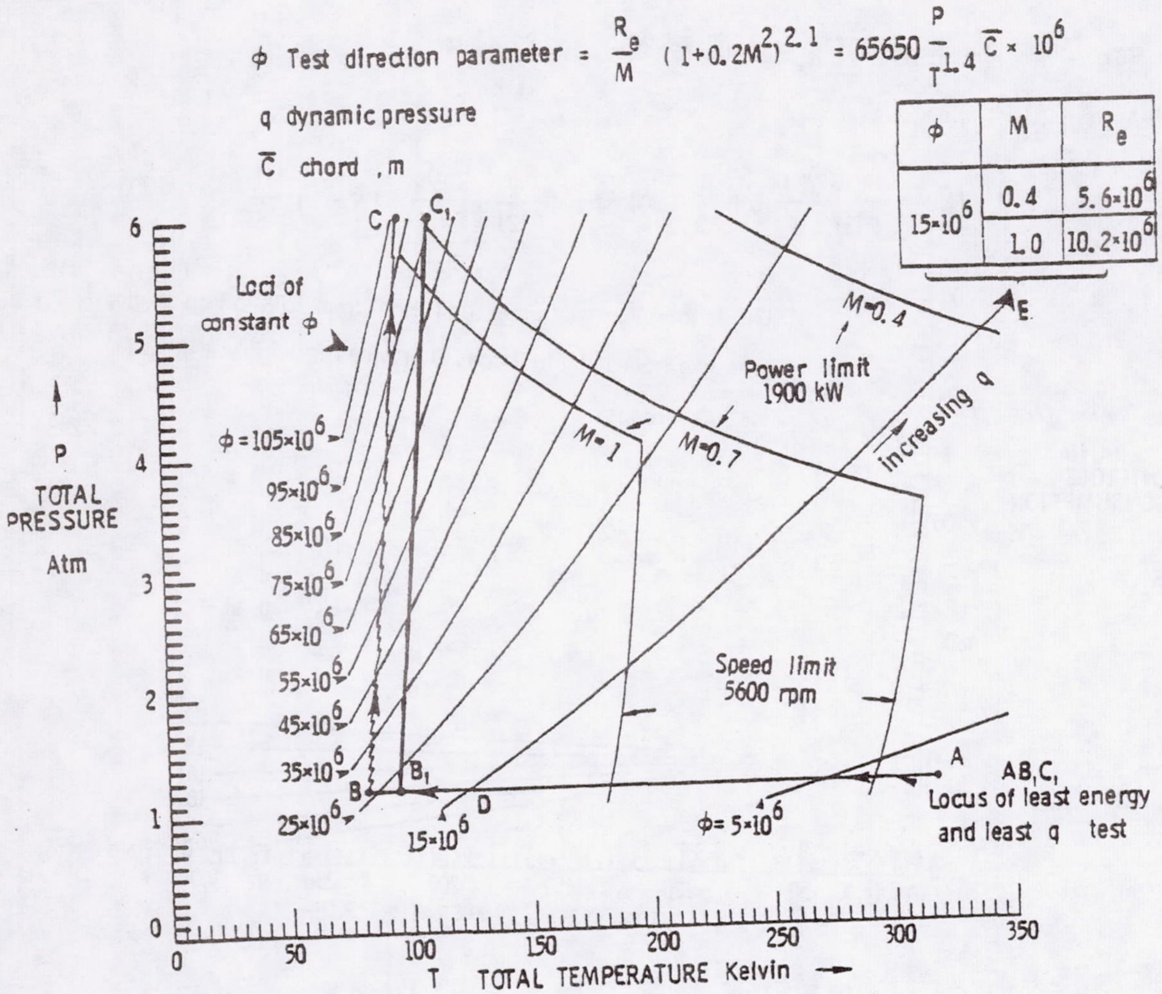

Figure 4. Test direction design in cryogenic wind tunnels—locus of least liquid nitrogen consumption and least q in P-T plane.

$R_e = 10\times10^6 \quad M = 0.6 \quad D = 2, 22, 4$ mts

$$LNC = \frac{\text{Fan power}}{(121 + T)}\,60D + (300 - T)\,\frac{W_t\,c_m + W_g\,c_v}{(121 + T)} + (P - 1)\,\frac{341\,V}{T}$$

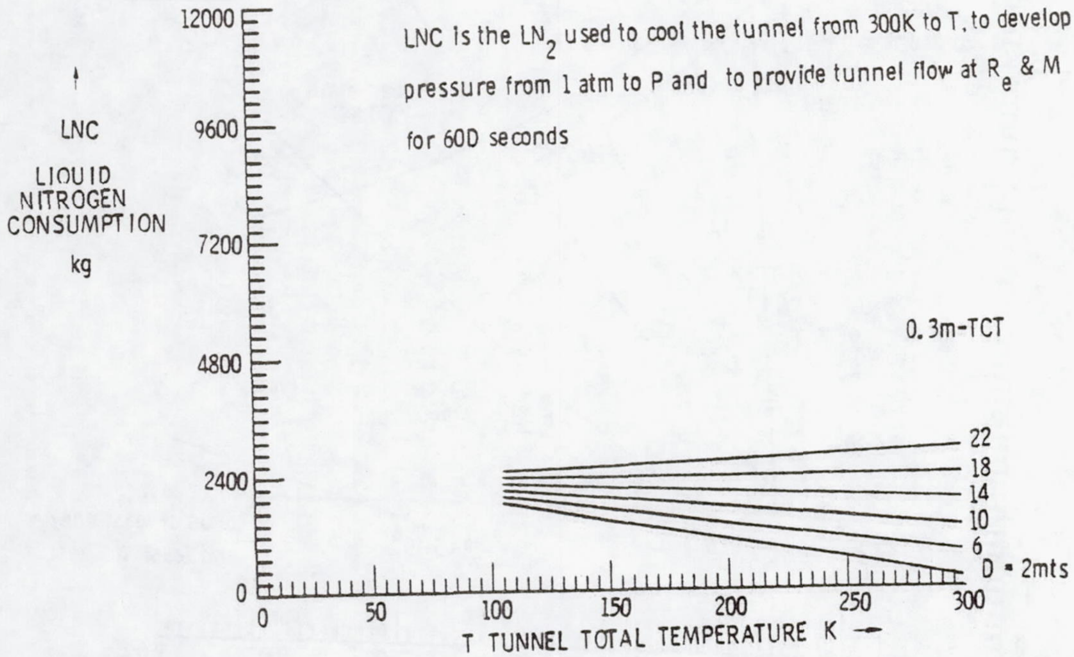

Figure 5. Liquid nitrogen consumption in a cryogenic tunnel test at $R_e = 10 \times 10^6$, M = 0.6, and D = 2 to 22 in 4-min steps.

$$R_e = 20\ 10^6 \quad M = 0.6 \quad D = 2, 22, 4 \text{ mts}$$

$$\text{LNC} = \frac{\text{Fan power}}{(121 + T)} 60\ D + (300 - T) \frac{W_t c_m + W_g c_v}{(121 + T)} + (P - 1) \frac{341\ V}{T}$$

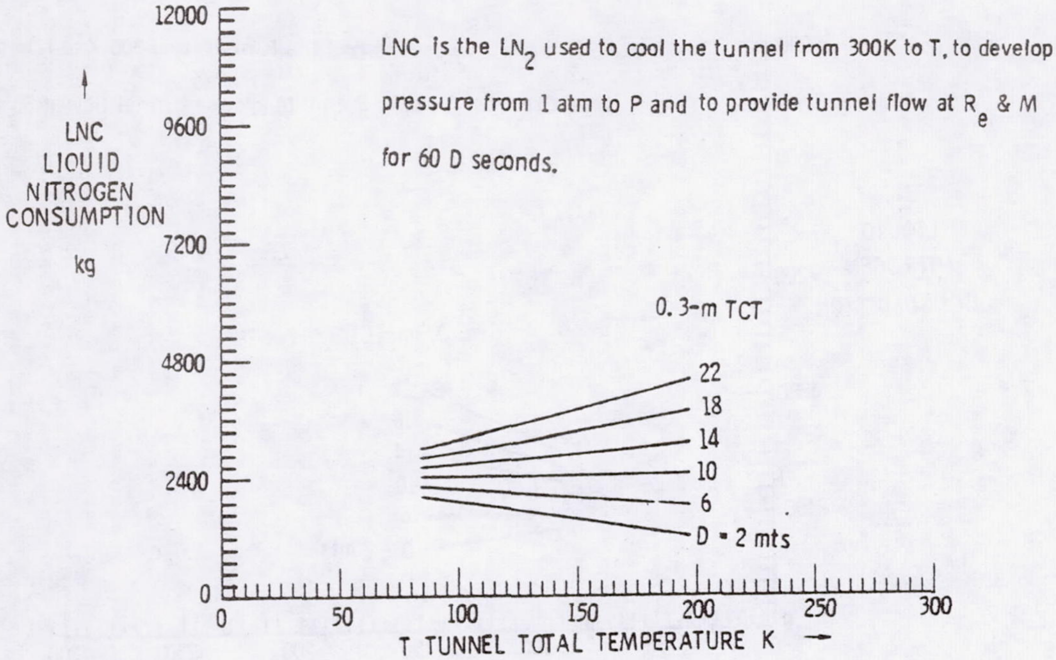

Figure 6. Liquid nitrogen consumption in a cryogenic tunnel test at $R_e = 20 \times 10^6$, M = 0.6, and D = 2 to 22 in 4-min steps.

$R_e = 30\ 10^6 \quad M = 0.6 \quad D = 2, 22, 4$ mts

$$LNC = \frac{\text{Fan power}}{(121 + T)} 60\ D + (300 - T)\frac{W_t c_m + W_g c_v}{(121 + T)} + (P - 1)\frac{341\ V}{T}$$

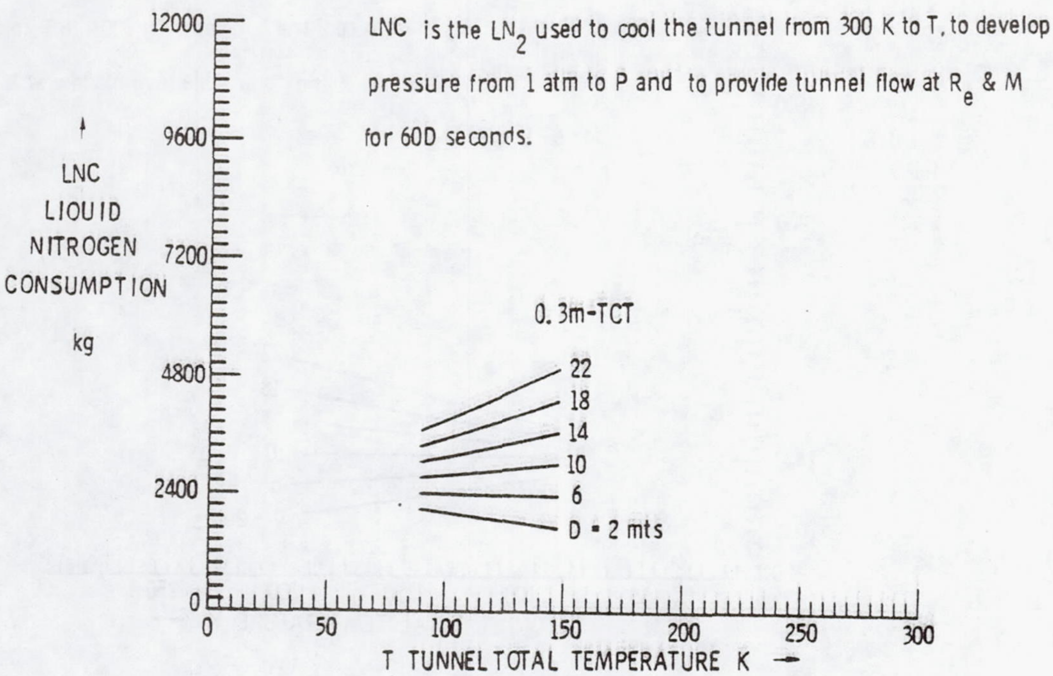

Figure 7. Liquid nitrogen consumption in a cryogenic tunnel test at $R_e = 30 \times 10^6$, $M = 0.6$, and $D = 2$ to 22 in 4-min steps.

$$R_e = 40 \; 10^6 \quad M = 0.6 \quad D = 2, 22, 4 \; \text{mts}$$

$$\text{LNC} = \frac{\text{Fan power}}{(121 + T)} \, 60\,D + (300 - T)\frac{W_t\, c_m + W_g\, c_v}{(121 + T)} + (P - 1)\,\frac{341\,V}{T}$$

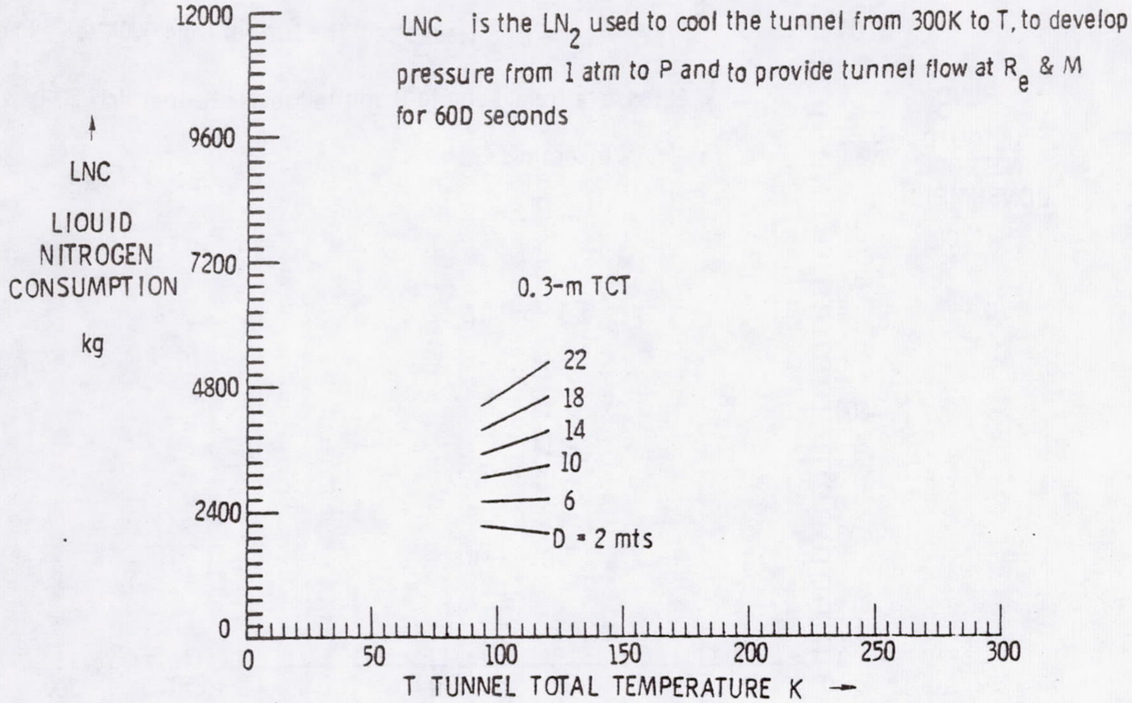

Figure 8. Liquid nitrogen consumption in a cryogenic tunnel test at $R_e = 40 \times 10^6$, M = 0.6, and D = 2 to 22 in 4-min steps.

$R_e = 50\ 10^6 \quad M = 0.6 \quad D = 2, 22, 4$ mts

$$LNC = \frac{\text{Fan power}}{(121 + T)} 60\,D + (300 - T)\frac{W_t c_m + W_g c_v}{(121 + T)} + (P - 1)\frac{341\,V}{T}$$

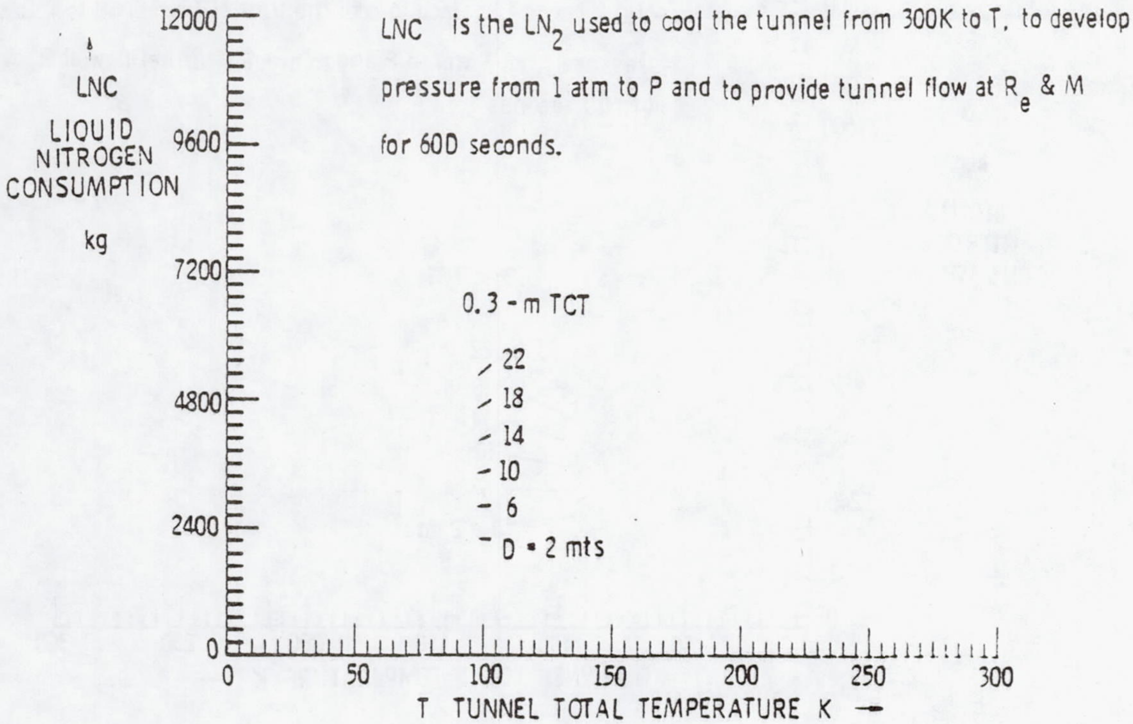

Figure 9. Liquid nitrogen consumption in a cryogenic tunnel test at $R_e = 50 \times 10^6$, $M = 0.6$, and $D = 2$ to 22 in 4-min steps.

$R_e = 10\ 10^6 \quad M = 0.9 \quad D = 2, 22, 4$ mts

$$LNC = \frac{\text{Fan power}}{(121 + T)} 60D + (300 - T)\frac{W_t c_m + W_g c_v}{(121 + T)} + (P - 1)\frac{341\,V}{T}$$

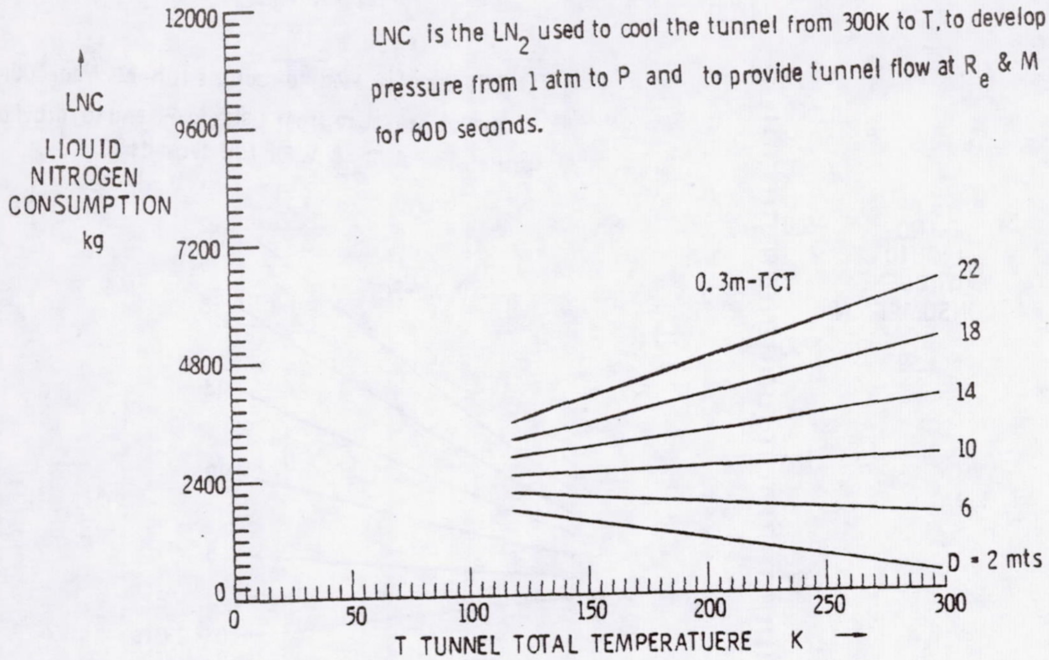

Figure 10. Liquid nitrogen consumption in a cryogenic tunnel test at $R_e = 10 \times 10^6$, M = 0.9, and D = 2 to 22 in 4-min steps.

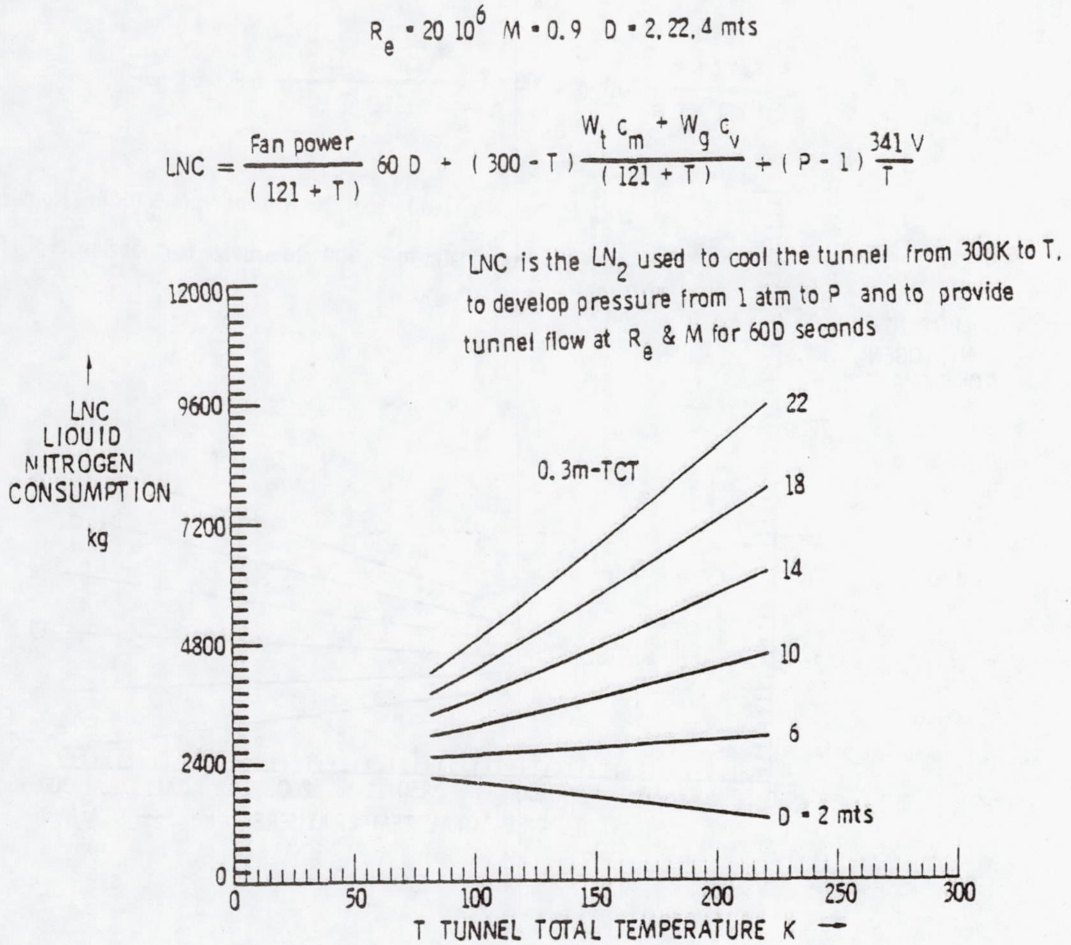

Figure 11. Liquid nitrogen consumption in a cryogenic tunnel test at $R_e = 20 \times 10^6$, M = 0.9, and D = 2 to 22 in 4-min steps.

$$R_e = 30\ 10^6 \quad M = 0.9 \quad D = 2, 22, 4 \text{ mts}$$

$$LNC = \frac{\text{Fan power}}{(121 + T)} 60\,D + (300 - T)\frac{W_t c_m + W_g c_v}{(121 + T)} + (P - 1)\,\frac{341\,V}{T}$$

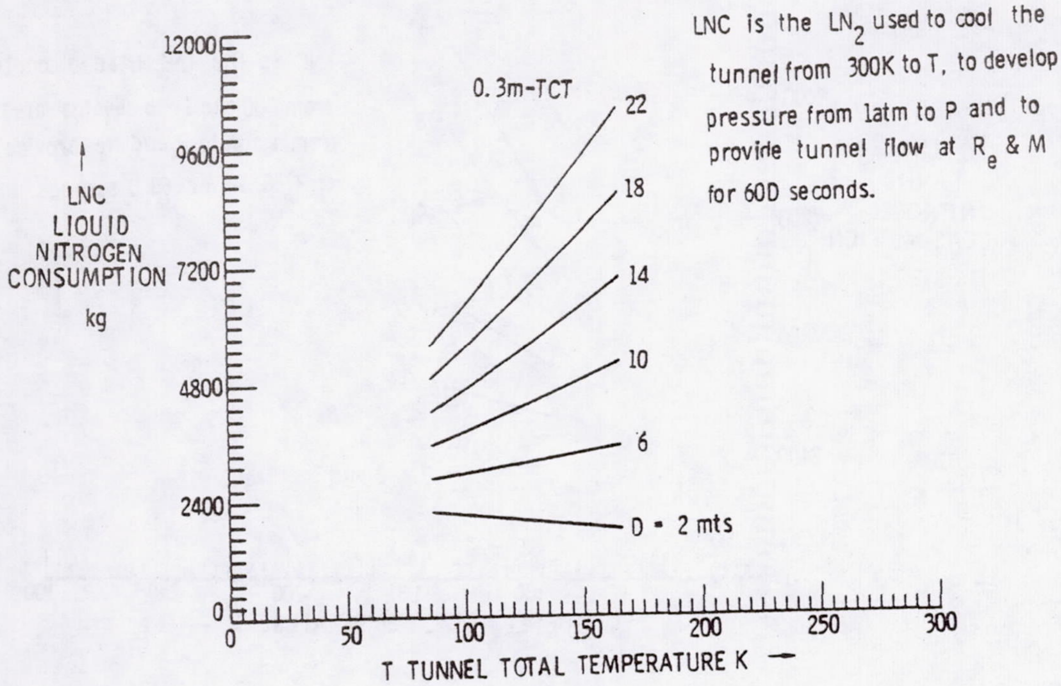

Figure 12. Liquid nitrogen consumption in a cryogenic tunnel test at $R_e = 30 \times 10^6$, M = 0.9, and D = 2 to 22 in 4-min steps.

$R_e = 40 \; 10^6 \qquad M = 0.9 \qquad D = 2, 22, 4 \text{ mts}$

$$LNC = \frac{\text{Fan power}}{(121 + T)} \, 60 \, D + (300 - T) \frac{W_t c_m + W_g c_v}{(121 + T)} + (P - 1) \frac{341 \, V}{T}$$

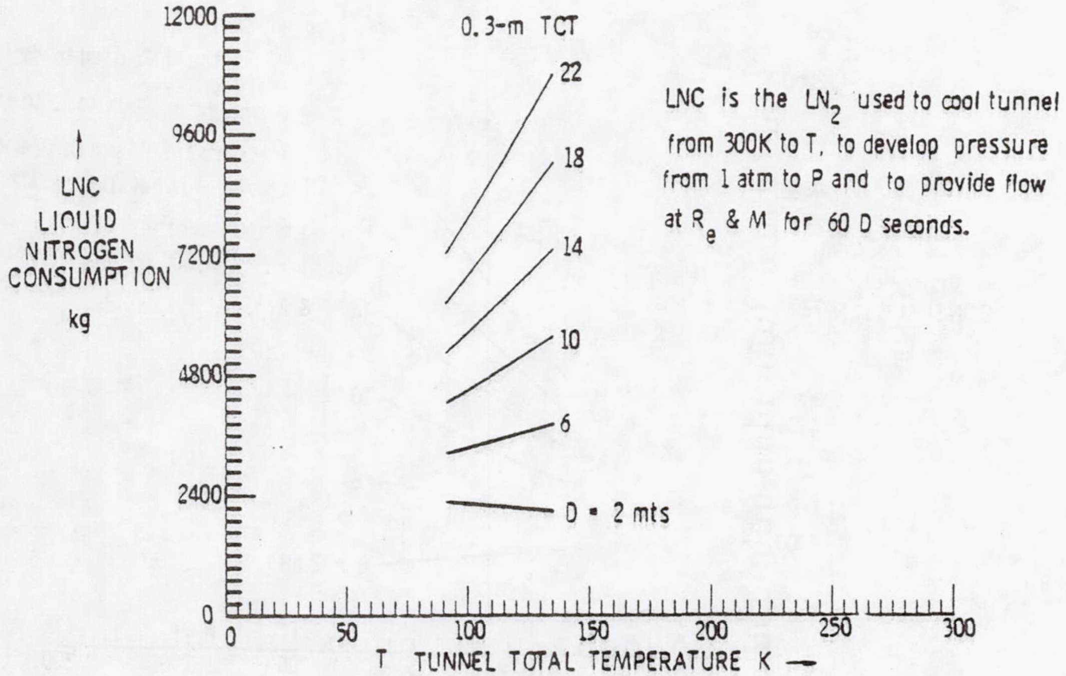

Figure 13. Liquid nitrogen consumption in a cryogenic tunnel test at $R_e = 40 \times 10^6$, M = 0.9, and D = 2 to 22 in 4-min steps.

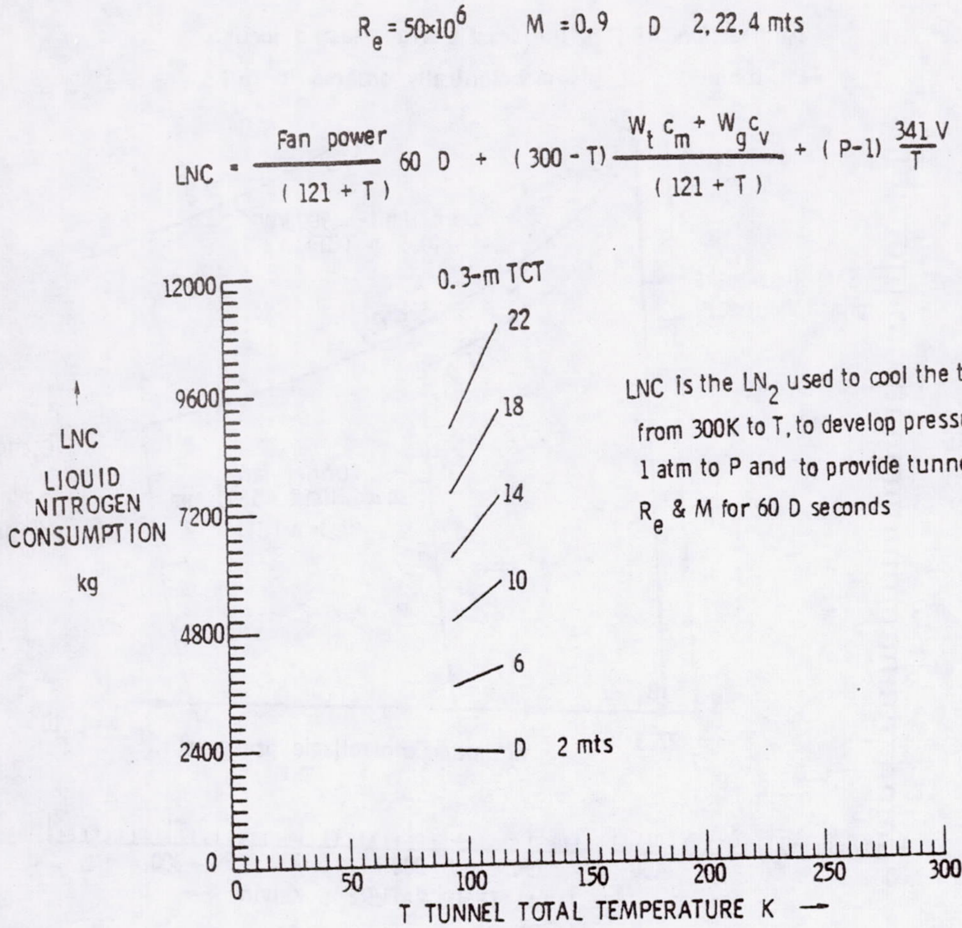

Figure 14. Liquid nitrogen consumption in a cryogenic tunnel test at $R_e = 50 \times 10^6$, M = 0.9, and D = 2 to 22 in 4-min steps.

$$\phi \text{ Test direction parameter} = \frac{R_e}{M}\left(1 + 0.2\,M^2\right)^{2.1} = 65650\,\frac{P}{T^{1.4}}\,\overline{C}\times 10^6$$

Test direction AB_1C_1 is the least energy least 'q' locus

Test condition set is monotonically ordered in ϕ $[\phi_1 \cdots \phi_i \cdots \phi_n]$

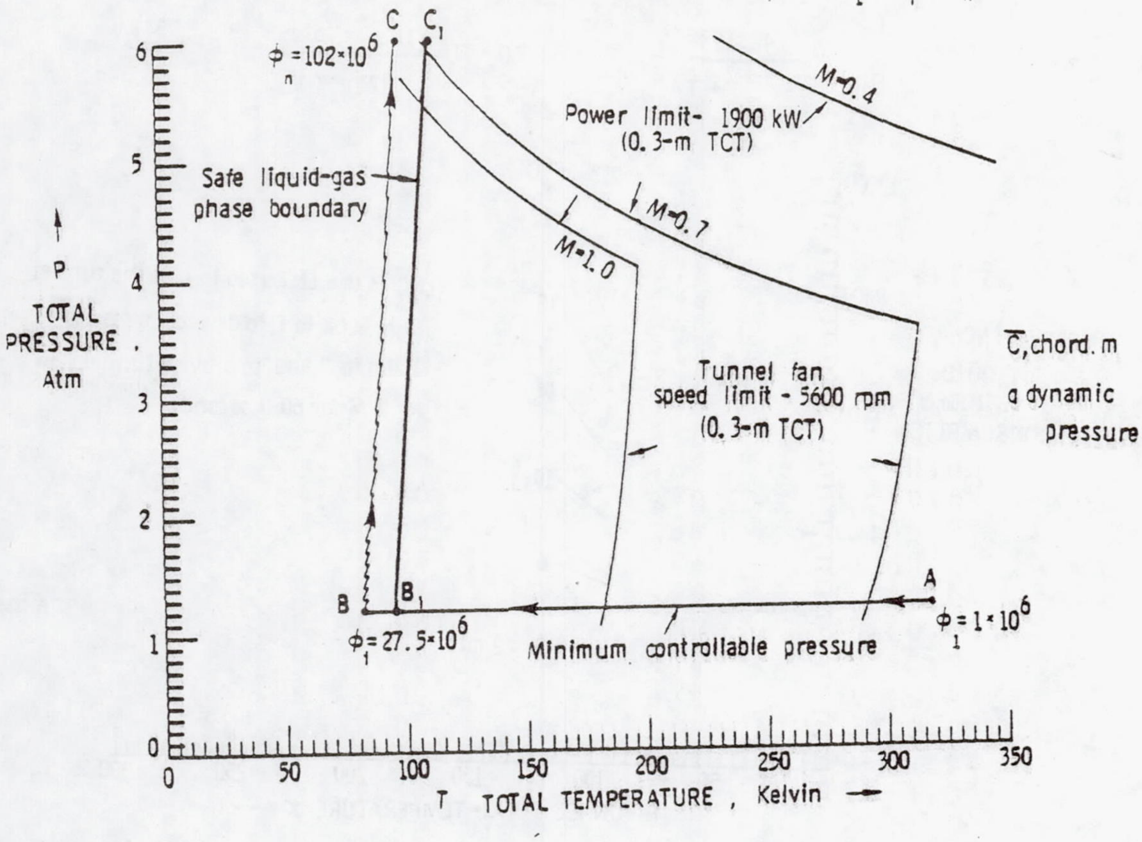

Figure 15. Test direction design—least energy consumption locus.

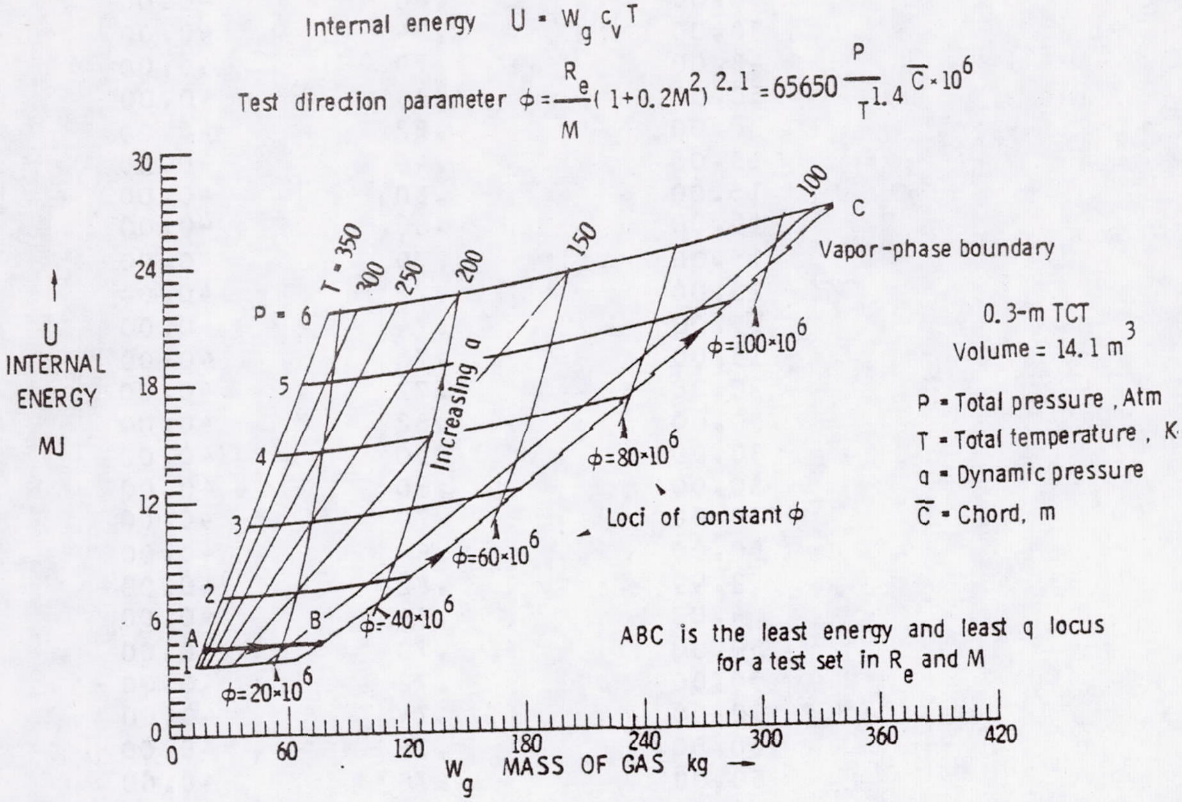

Figure 16. Energy state diagram and test direction locus in cryogenic wind tunnels.

RE	M	D
30.00	.74	40.00
30.00	.76	40.00
30.00	.78	40.00
30.00	.80	40.00
30.00	.84	40.00
48.00	.78	40.00
50.00	.80	40.00
30.00	.82	40.00
15.00	.40	40.00
15.00	.50	40.00
15.00	.60	40.00
15.00	.70	40.00
15.00	.72	40.00
15.00	.74	40.00
15.00	.76	40.00
30.00	.72	40.00
50.00	.82	40.00
30.00	.50	40.00
30.00	.60	40.00
30.00	.70	40.00
50.00	.84	40.00
3.99	.82	40.00
4.03	.84	40.00
50.00	.70	40.00
50.00	.72	40.00
50.00	.74	40.00
50.00	.76	40.00
50.00	.78	40.00
2.18	.40	40.00
.90	.10	1.00
2.60	.50	45.00
3.02	.60	40.00
3.44	.70	40.00
3.52	.72	40.00
3.61	.74	40.00
3.69	.76	40.00
3.78	.78	40.00
3.86	.80	40.00

Figure 17. A cryogenic tunnel test program of 38 flow parameter conditions, R_e-M-D.

DIRECTION	RE	M
5.76	2.60	.50
5.82	2.18	.40
5.82	3.02	.60
5.98	3.44	.70
6.02	3.52	.72
6.06	3.61	.74
6.11	3.69	.76
6.16	3.78	.78
6.21	3.86	.80
6.33	4.03	.84
6.35	3.99	.82
9.04	.90	.10
24.83	15.00	.76
25.21	15.00	.74
25.63	15.00	.72
26.08	15.00	.70
28.93	15.00	.60
33.24	15.00	.50
40.06	15.00	.40
47.12	30.00	.84
47.68	30.00	.82
48.29	30.00	.80
48.95	30.00	.78
49.66	30.00	.76
50.43	30.00	.74
51.26	30.00	.72
52.15	30.00	.70
57.86	30.00	.60
66.47	30.00	.50
78.32	48.00	.78
78.54	50.00	.84
79.47	50.00	.82
80.49	50.00	.80
81.58	50.00	.78
82.77	50.00	.76
84.05	50.00	.74
85.43	50.00	.72
86.92	50.00	.70

Figure 18. Order of execution of the cryogenic tunnel test program.

MARGIN TO ONSET OF CONDENSATION,K
12.0

RE	M	P	T	DWELL	FANLN	TLN	DEP	DET
2.60	.50	1.20	235.02	45.0	861.7	588.9	-.09	-.84
2.18	.40	1.20	233.24	40.0	413.6	16.0	-.09	-.83
3.02	.60	1.20	233.19	40.0	1245.3	.5	-.09	-.83
3.44	.70	1.20	228.83	40.0	1645.8	39.2	-.09	-.81
3.52	.72	1.20	227.72	40.0	1979.2	10.0	-.09	-.81
3.61	.74	1.20	226.55	40.0	2116.6	10.6	-.09	-.81
3.69	.76	1.20	225.31	40.0	2258.0	11.1	-.09	-.80
3.78	.78	1.20	224.01	40.0	2403.1	11.7	-.09	-.80
3.86	.80	1.20	222.66	40.0	2551.7	12.2	-.09	-.79
4.03	.84	1.20	219.80	40.0	2858.6	25.7	-.09	-.78
3.99	.82	1.20	219.27	40.0	2707.1	4.7	-.09	-.78
.90	.10	1.20	170.43	1.0	.2	439.6	-.09	-.61
15.00	.76	1.37	92.05	40.0	2678.4	714.2	-.10	-.33
15.00	.74	1.40	92.47	40.0	2568.6	1.6	-.10	-.33
15.00	.72	1.42	92.36	40.0	2437.8	2.1	-.11	-.33
15.00	.70	1.45	92.61	40.0	2323.6	1.5	-.11	-.33
15.00	.60	1.63	93.60	40.0	1768.8	9.2	-.12	-.33
15.00	.50	1.91	95.12	40.0	1277.7	14.1	-.14	-.34
15.00	.40	2.37	97.37	40.0	856.1	22.6	-.18	-.35
30.00	.84	2.86	99.45	40.0	7084.5	23.5	-.22	-.35
30.00	.82	2.90	99.62	40.0	6801.2	1.9	-.22	-.35
30.00	.80	2.94	99.72	40.0	6514.0	1.9	-.22	-.35
30.00	.78	2.99	99.98	40.0	6245.4	2.4	-.23	-.36
30.00	.76	3.04	100.16	40.0	5972.1	2.4	-.23	-.36
30.00	.74	3.09	100.27	40.0	5695.3	2.4	-.23	-.36
30.00	.72	3.15	100.50	40.0	5433.7	2.9	-.24	-.36
30.00	.70	3.21	100.65	40.0	5168.2	2.8	-.24	-.36
30.00	.60	3.62	102.06	40.0	3946.3	19.2	-.28	-.36
30.00	.50	4.25	103.99	40.0	2855.4	29.0	-.32	-.37
48.00	.78	5.14	106.40	40.0	10762.9	40.0	-.40	-.38
50.00	.84	5.16	106.49	40.0	12817.2	.9	-.40	-.38
50.00	.82	5.23	106.65	40.0	12299.1	3.1	-.40	-.38
50.00	.80	5.31	106.87	40.0	11797.4	3.6	-.41	-.38
50.00	.78	5.39	107.03	40.0	11288.4	3.6	-.42	-.38
50.00	.76	5.48	107.24	40.0	10794.0	4.0	-.42	-.38
50.00	.74	5.58	107.50	40.0	10312.2	4.4	-.43	-.38
50.00	.72	5.69	107.79	40.0	9841.0	4.9	-.44	-.38
50.00	.70	5.81	108.11	40.0	9379.1	5.3	-.45	-.38

TOTAL LN2 USED
192252.535

Figure 19. Minimum energy test direction design for the cryogenic tunnel test program.